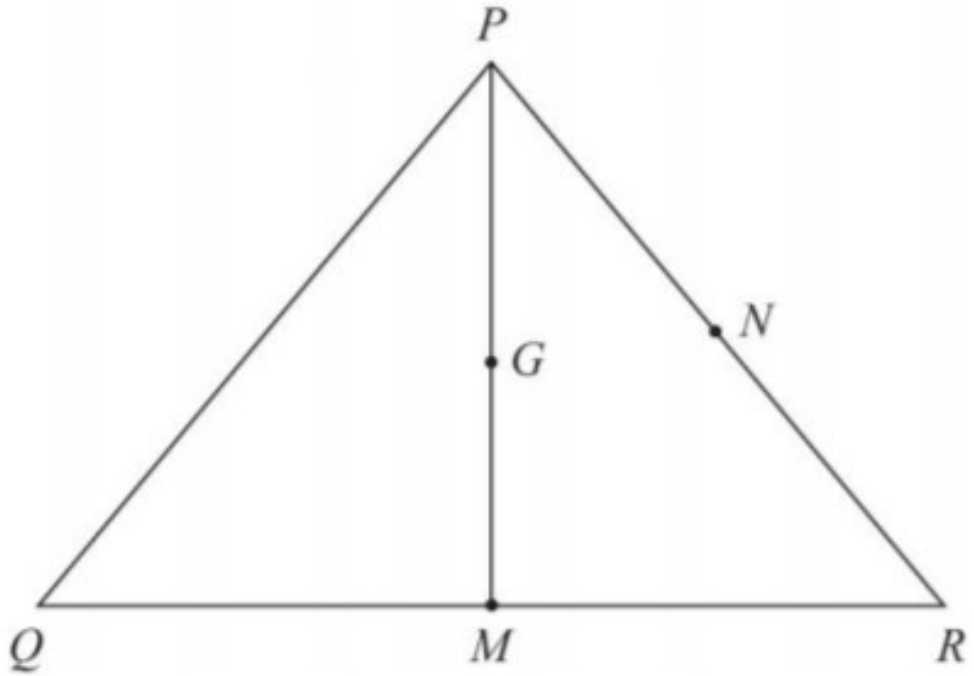
Starter Question

M is the midpoint of QR. N is the midpoint of PR

$\overrightarrow{PQ} = \mathbf{b}$ and $\overrightarrow{PR} = \mathbf{a}$



Work out $\overrightarrow{PM}$ and $\overrightarrow{QN}$ in terms of **a** and **b**

J Vectors

J1 Use vectors in two dimensions and in three dimensions.

Assessed at AS and A-level

Teaching guidance

Students should:

- become familiar with both column vectors and **i**, **j** notation, where **i** and **j** are unit vectors in perpendicular directions. Unless specified otherwise, students may use either notation in their solutions

  Note: questions may be set without context as pure vector questions, or in context, for example, using vectors to represent velocities or forces.

- know that vectors may be used to describe translations of graphs.

J Vectors

J2

Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

Assessed at AS and A-level

Teaching guidance

Students should be able to answer questions using vectors in two dimensions.

6.2 Components of a vector

Vectors can be expressed in component form. This makes addition, subtraction and multiplying by a scalar quantity much easier.

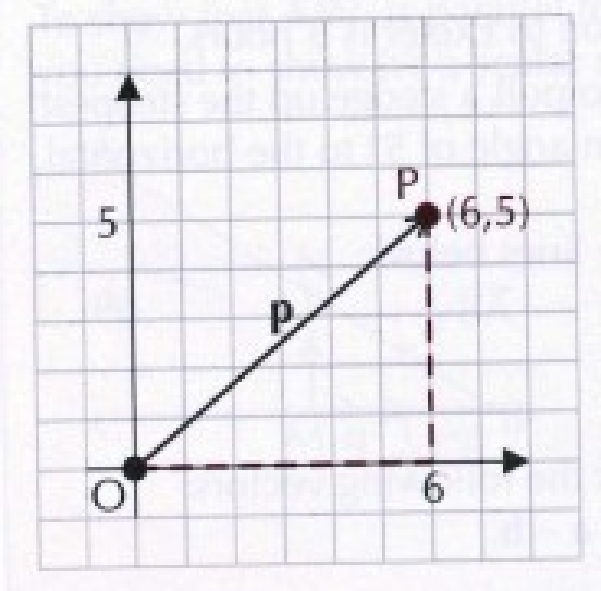
Vectors written in component form can be written in two ways:

1. and notation
2. Column vectors

6.2 Components of a vector

Position Vectors

You can use a vector to describe the position of a point in relation to the **origin**, O. This is called a position vector.



We can describe point P using its Cartesian coordinates (6, 5).

We can also describe how far it is from the origin using the position vector which has a horizontal and a vertical component: .

6.2 Components of a vector

The horizontal component is represented by .

The vertical component is represented by .

is a unit vector in the positive x-direction

$|\square| = 1$

$|\square| = 1$

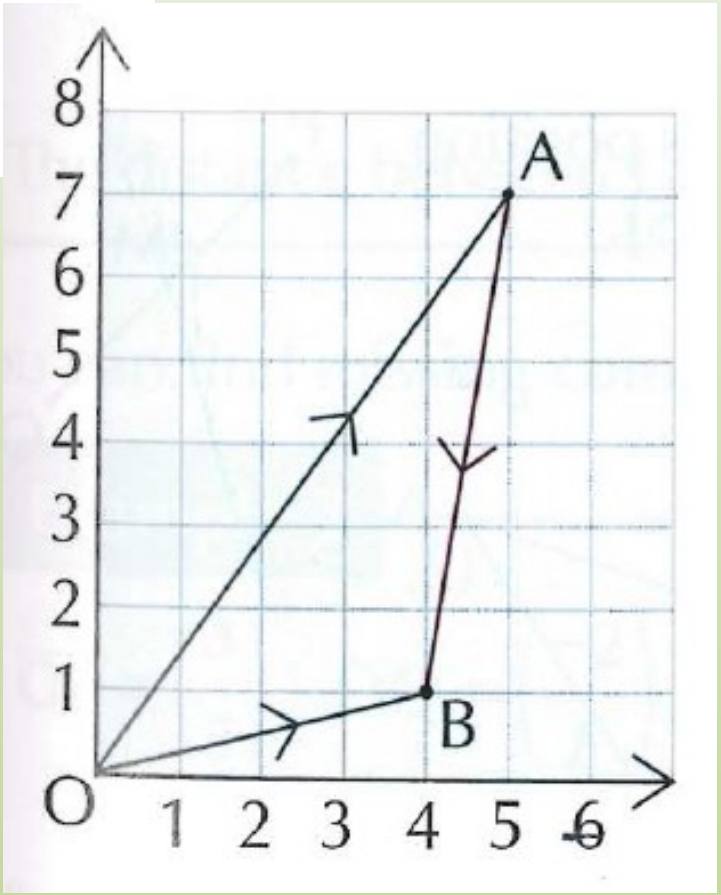
is a unit vector in the positive y-direction

6.2 Components of a vector

Example 1

a) Write down the position vectors of A and B in i + j form.

b) Hence find $\overrightarrow{AB}$ in terms of i and j.

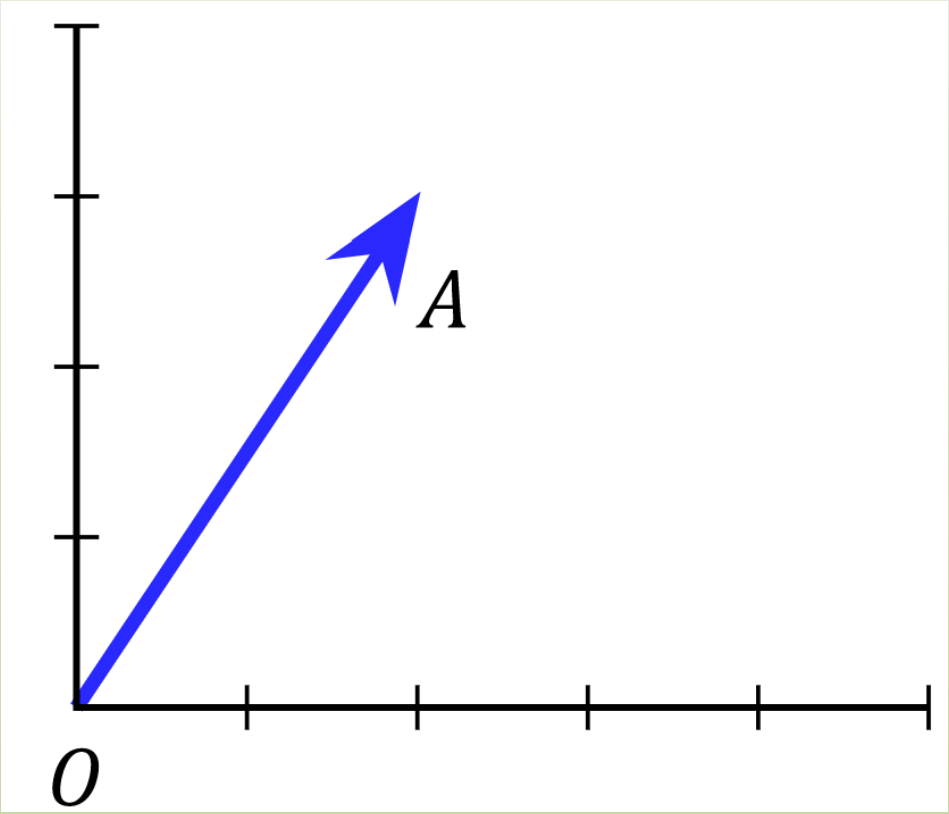


$\overrightarrow{OA} = 5\,\mathbf{i} + 7\,\mathbf{j}$

$\overrightarrow{OB} = 4\,\mathbf{i} + \mathbf{j}$

6.2 Components of a vector

Consider the following vector :

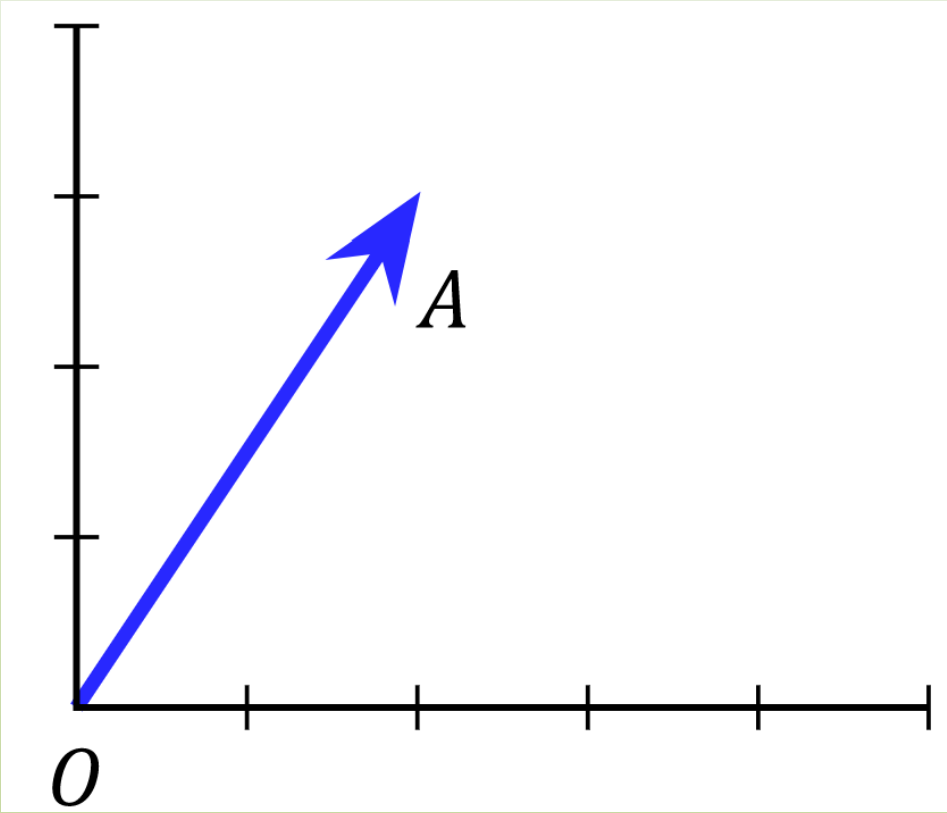


$= 2\mathbf{i} + 3\mathbf{j}$
in **i, j** notation

Or $=$
in column vector notation

6.2 Components of a vector

Consider the magnitude and direction of :



can be found using Pythagoras' Theorem:

=

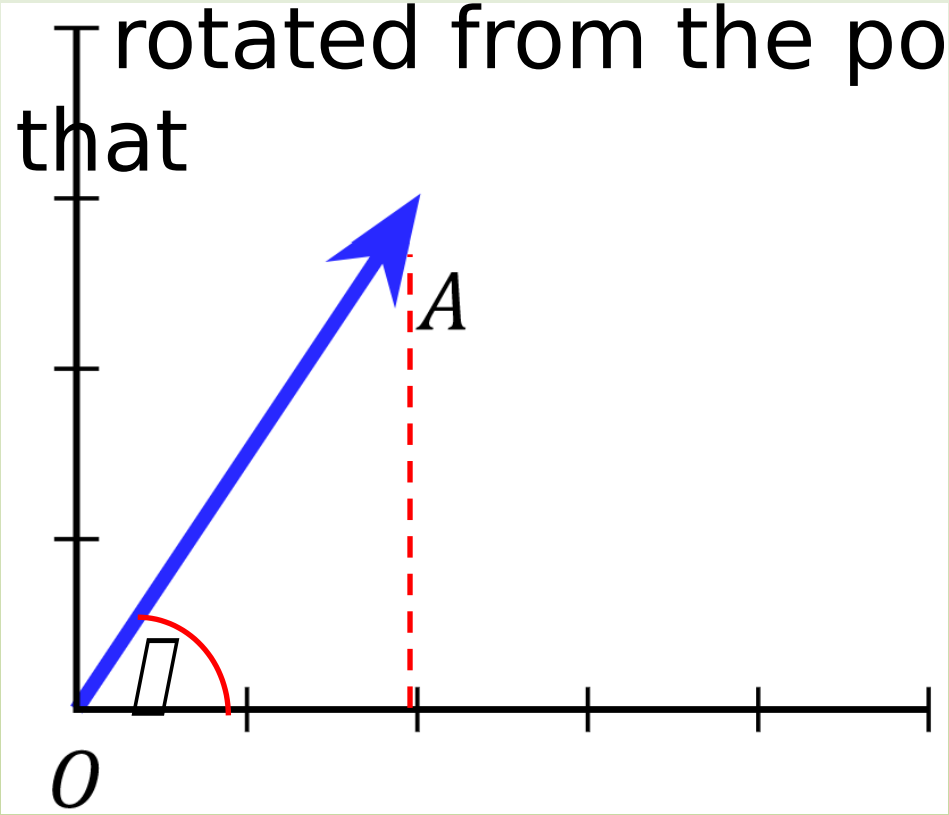
=

= 3.61 (3sf)

6.2 Components of a vector

The direction of would be quoted as an angle, ,

 rotated from the positive x-axis, such that



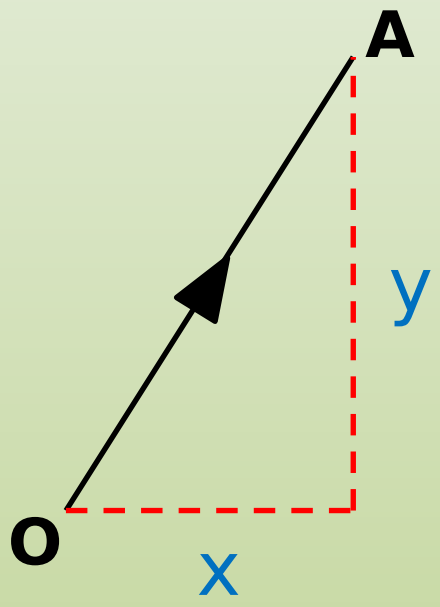
=

(1dp)

6.2 Components of a vector

In general:

If then:



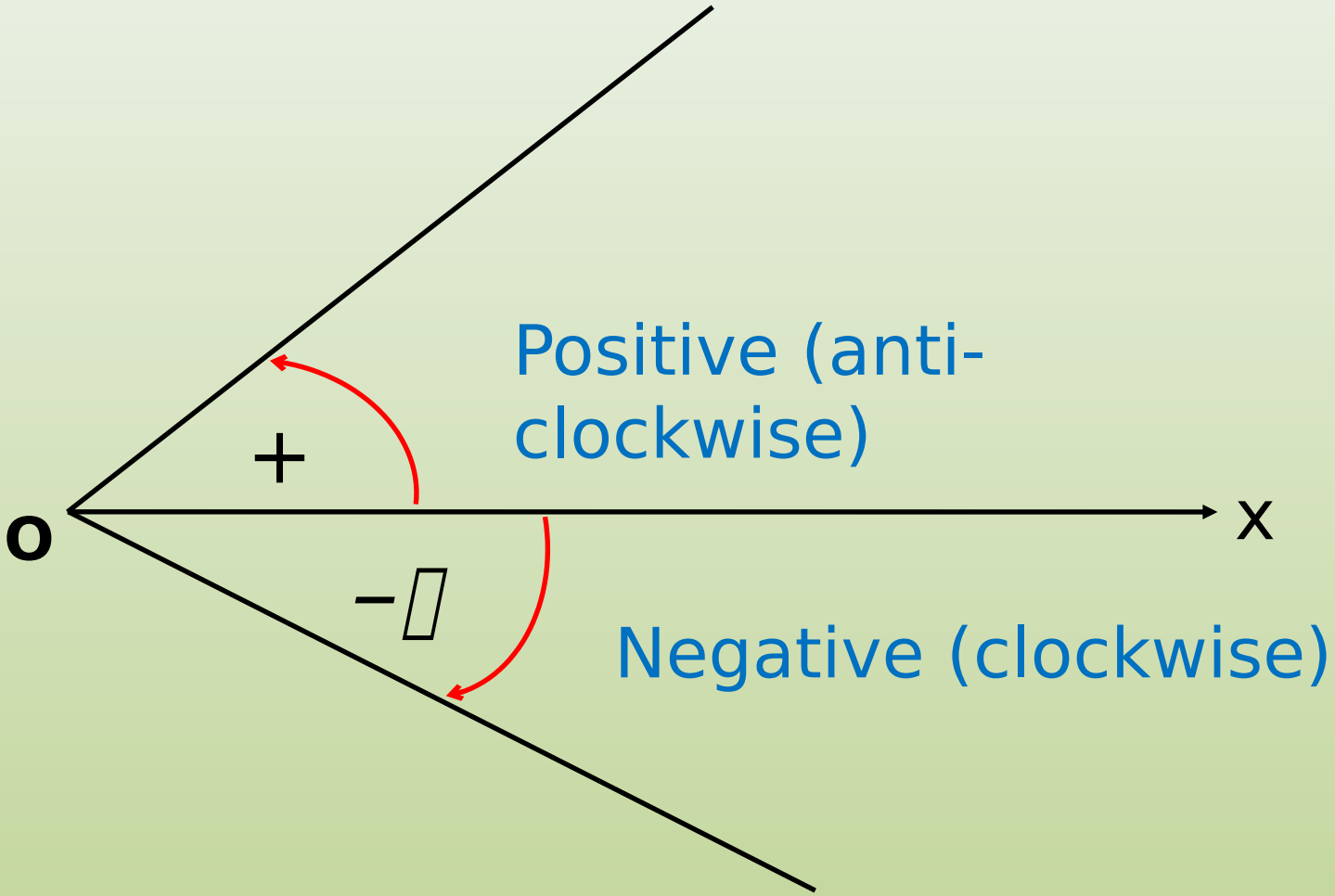
$$=$$

$$=$$

$$-180<\theta\leq 180$$

6.2 Components of a vector

NB:



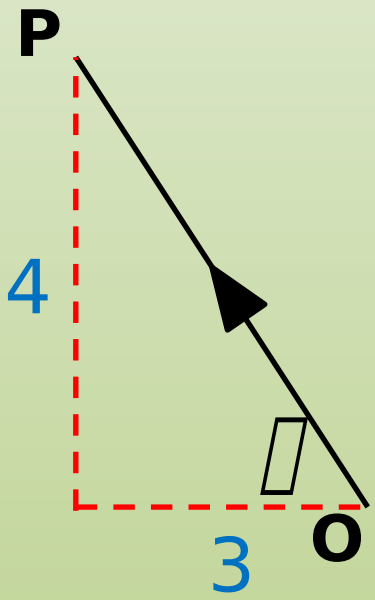
6.2 Components of a vector

Example 2

Work out the magnitude and direction of the following vectors:

a) p = -3i + 4j

$$|\square| = \sqrt{(-3)^2 + (4)^2} = 5$$



Direction:

direction is to the positive x-axis

6.2 Components of a vector

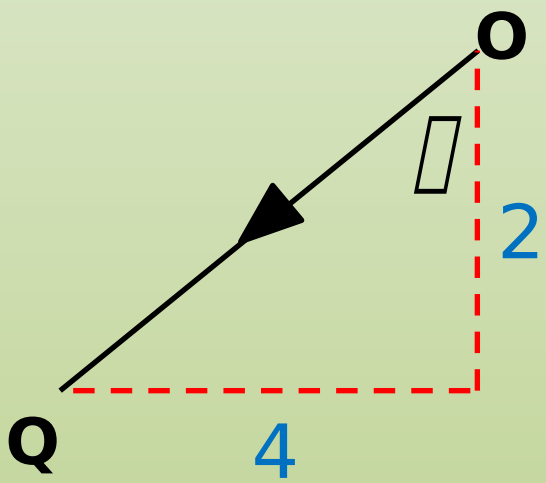
Example 2

Work out the magnitude and direction of the following vectors:

a) p = -3i + 4j

$$|\square| = \sqrt{(-4)^2 + (-2)^2} = 2\sqrt{5}$$

b) q =



Direction:

direction is **so -153.4 to the positive x-axis**

6.2 Components of a vector

Example 3

$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ b \end{pmatrix}$, given that $|\overrightarrow{PQ}| = \sqrt{29}$ and $|\overrightarrow{OQ}| = \sqrt{13}$, find b.

6.2 Components of a vector

Example 3

$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $\overrightarrow{OQ} = \begin{pmatrix} -2 \\ b \end{pmatrix}$, given that $|\overrightarrow{PQ}| = \sqrt{29}$ and $|\overrightarrow{OQ}| = \sqrt{13}$, find b.

From calc:

6.2 Components of a vector

Example 4

Find the unit vector in the direction of $\mathbf{q} = 5\mathbf{i} - 12\mathbf{j}$.

t vector... the vector divided by its magnitude...

$$|\square| = \sqrt{5^2 + 12^2} = 13$$

$$\therefore \hat{q} = \frac{5}{13}\square - \frac{12}{13}\square$$

6.2 Components of a vector

Finding the angle between two vectors

The angle between two vectors a and b can be calculated by constructing a triangle with a and b as two of its sides.

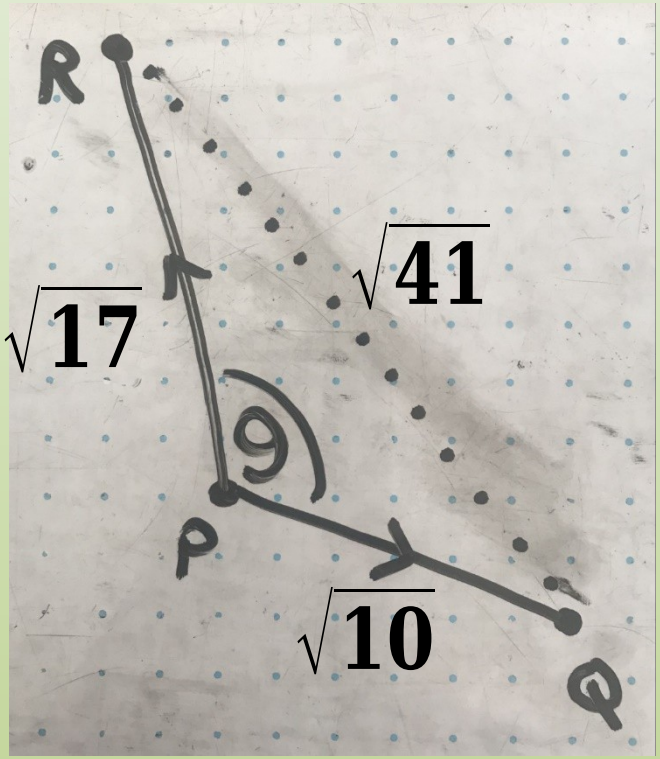
We can calculate the magnitude of these vectors and then use the cosine rule to find the angle between them.

6.2 Components of a vector

Example 5

Find the angle θ between the vectors $\overrightarrow{PQ} = 3\mathbf{i} - \mathbf{j}$ and $\overrightarrow{PR} = -\mathbf{i} + 4\mathbf{j}$.

Draw a diagram!!!



=

=

$$\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

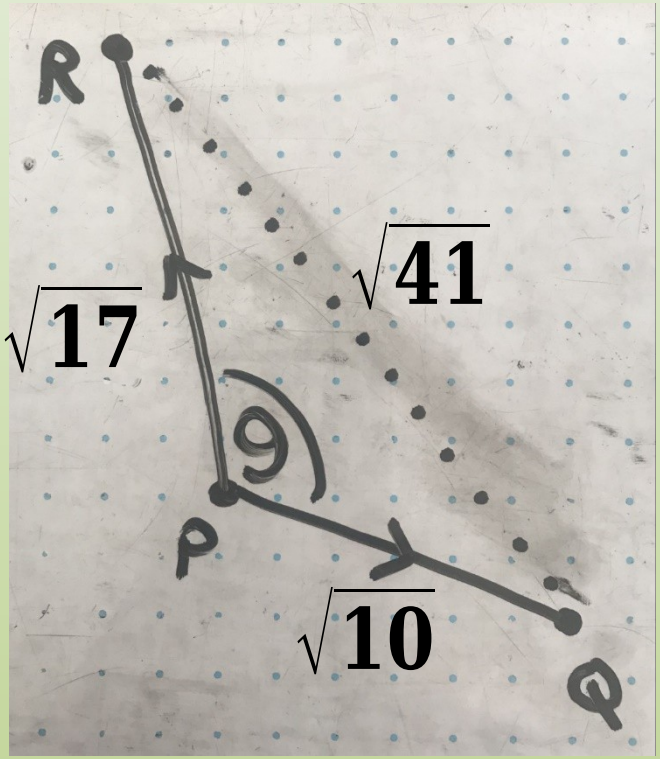
=

6.2 Components of a vector

Example 5

Find the angle θ between the vectors $\overrightarrow{PQ} = 3\mathbf{i} - \mathbf{j}$ and $\overrightarrow{PR} = -\mathbf{i} + 4\mathbf{j}$.

Draw a diagram!!!



$$\cos\theta = -\frac{14}{2\sqrt{10}\sqrt{17}} = -0.536\ldots$$

to 3sf

Note: you can find the angle between two vectors on the calculator in menu 5!

CGP Exercise 2.1

6.2 Components of a vector

Exam Question

Given that point A has the position vector $4\mathbf{i} + 7\mathbf{j}$ and point B has the position vector $10\mathbf{i} + q\mathbf{j}$, where q is a constant, find

a the vector $\overrightarrow{AB}$ in terms of q **(2 marks)**

b Given further that $\left|\overrightarrow{AB}\right| = 2\sqrt{13}$, find the two possible values of q showing detailed reasoning in your working. **(5 marks)**

4a	Makes an attempt to find the vector $\overrightarrow{AB}$ .For example, writing $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or $\overrightarrow{AB} = 10\mathbf{i} + q\mathbf{j} - (4\mathbf{i} + 7\mathbf{j})$	**M1**
	Shows a fully simplified answer: $\overrightarrow{AB} = 6\mathbf{i} + (q-7)\mathbf{j}$	**A1**
		(2)
4b	Correctly interprets the meaning of $\left\|\overrightarrow{AB}\right| = 2\sqrt{13}$, by writing $(6)^2 + (q-7)^2 = \left(2\sqrt{13}\right)^2$ o.e.	**M1**
	Correct method to solve quadratic equation in q (full working must be shown). For example, $(q-7)^2 = 16$ or $q^2 - 14q + 33 = 0$	**M1**
	$q - 7 = \pm 4$ or $(q-11)(q-3) = 0$ or $q = \dfrac{14 \pm \sqrt{14^2 - 4\times 1\times 33}}{2\times 1}$	**M1**
	$q = 11$	**A1**
	$q = 3$	**A1**
		(5)